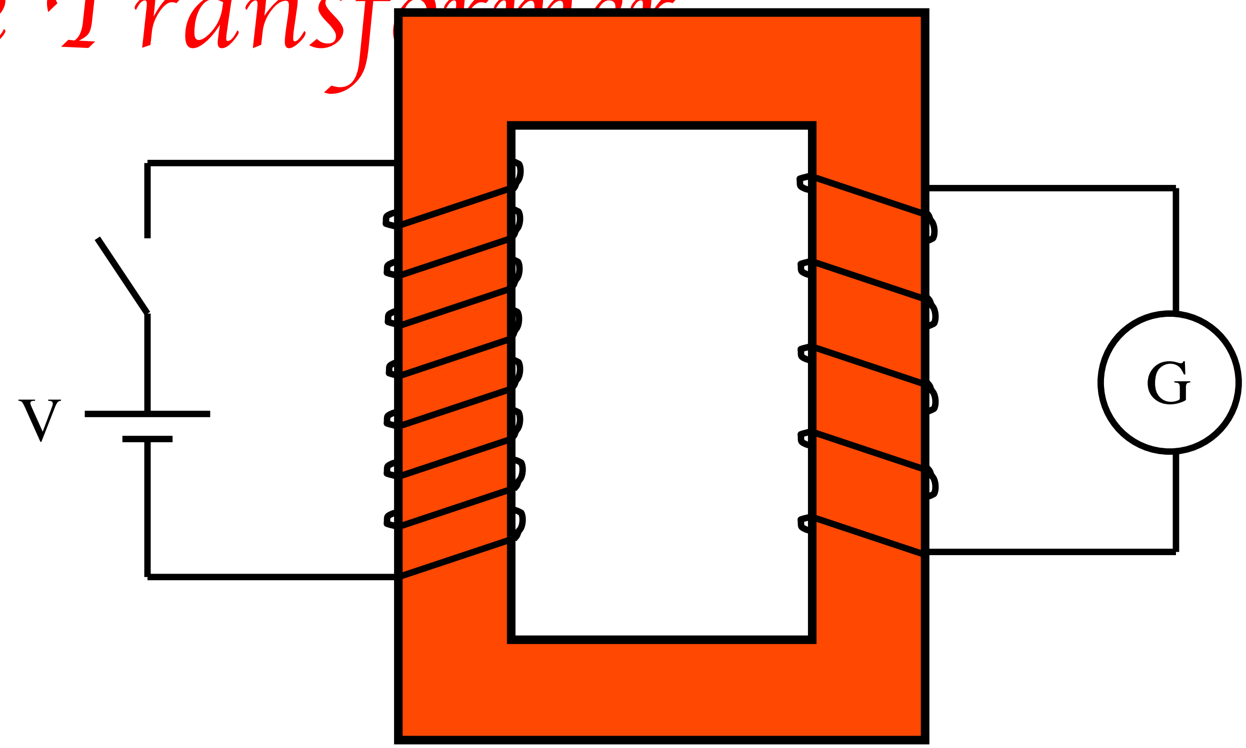
AP Ideas (Induction) with Non-AP Math
The Transformer

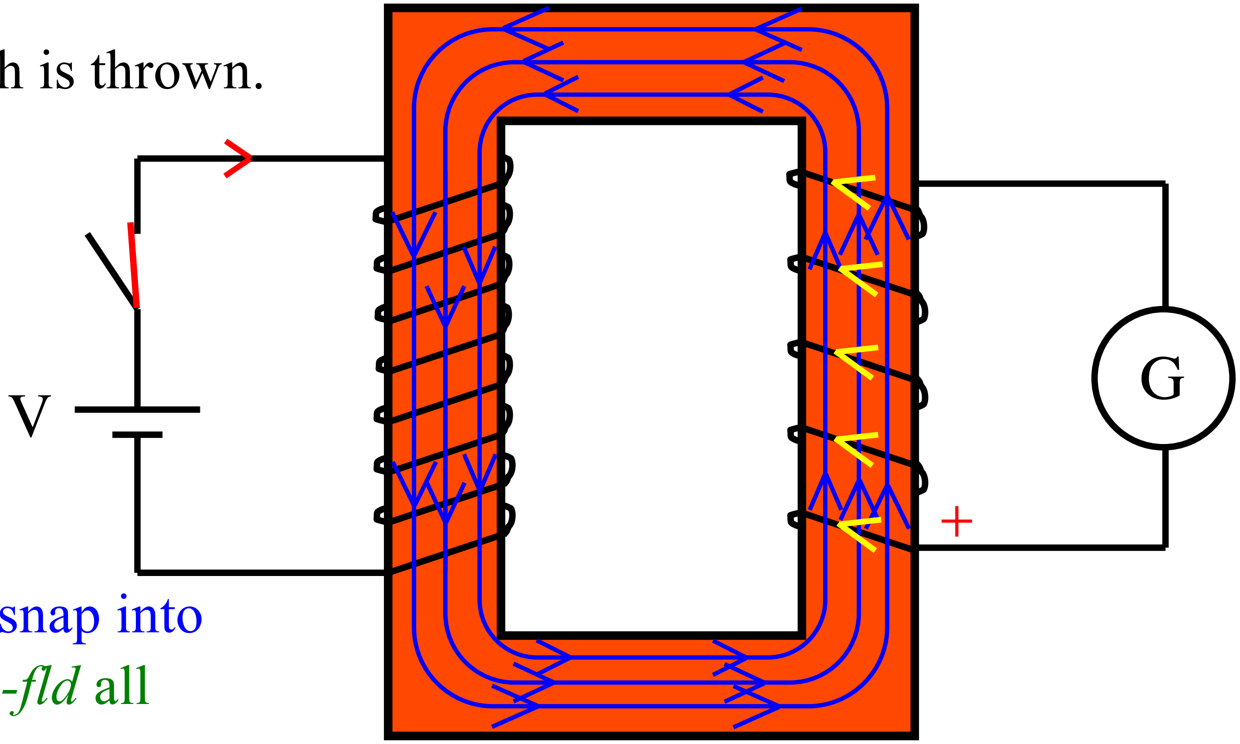
To the right you see an iron yoke about which is wrapped two independent coils that are insulated from one another (see sketch). To whit:



--The left circuit, called *the primary* because it includes a power supply in it, has some number of winds N_p in its coil.

--The right circuit, called *the secondary* because it includes a device you are transferring power to, has some number of winds N_s in its coil.

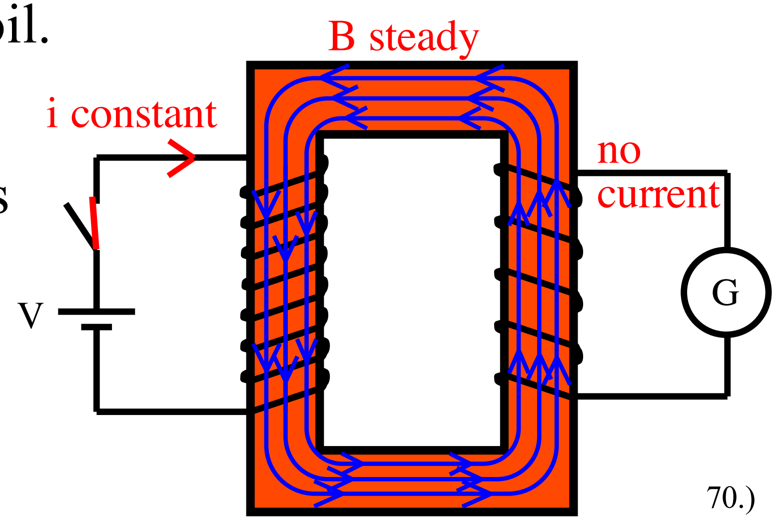
At some time, the switch is thrown.



--Current in the primary coil experiences a back EMF, but it slowly builds generating a slowly escalating *B-fld* down its axis.

--The domains in the yoke snap into alignment telescoping that *B-fld* all around the yoke.

--As the B-fld increases,an induced EMF is set up in the secondary coil producing an induced current in the secondary coil.

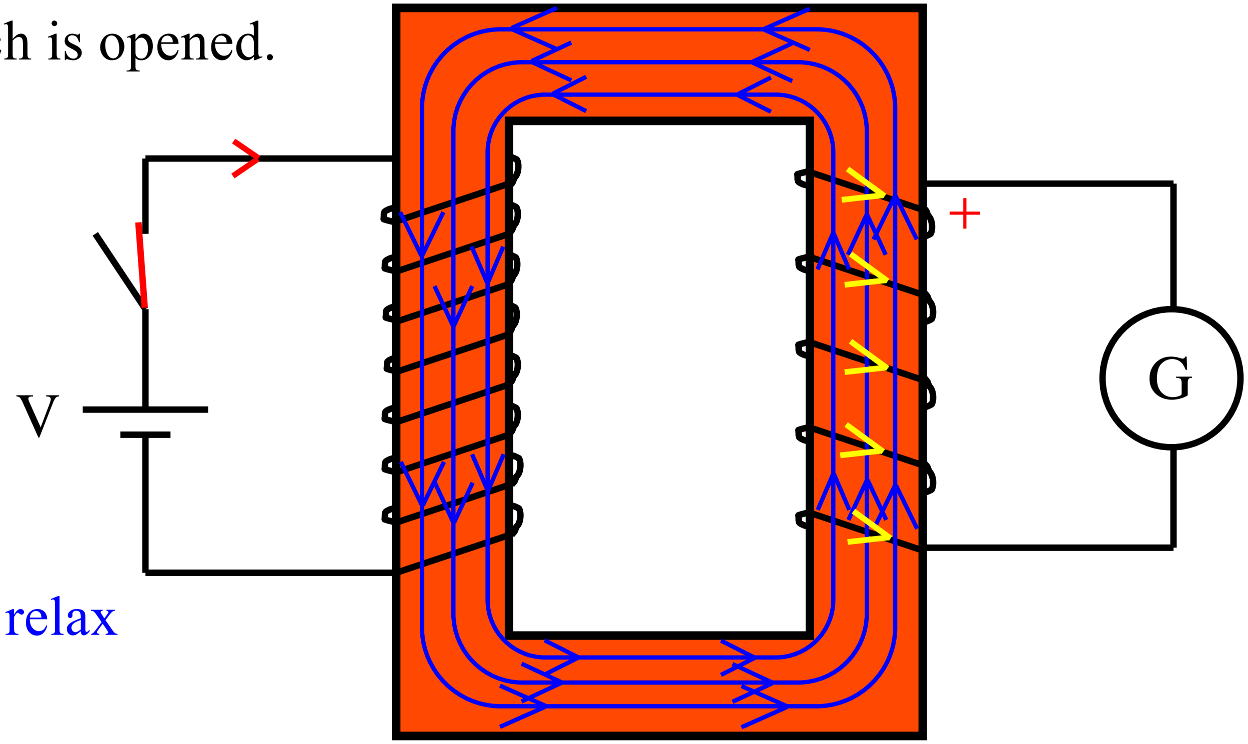


--The induced current in the secondary coil CONTINUES until the current in the primary has reached steady state whereupon the *B-fld* in the yoke ceases to change and there is no longer a *changing magnetic flux* through the secondary coil.

At some time, the switch is opened.

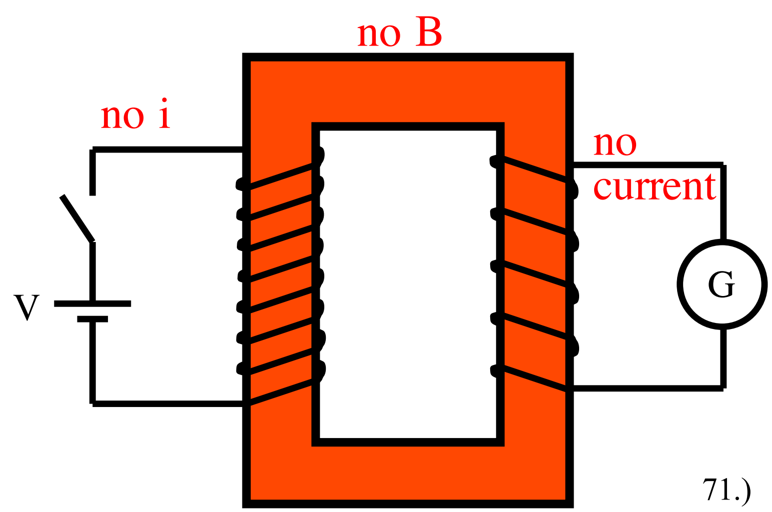
--As the current in the primary tries to drop, it (again) experiences an EMF which slows the diminishing as the *B-fld* down the axis erases.

--The domains in the yoke relax with the *B-fld* diminishing.

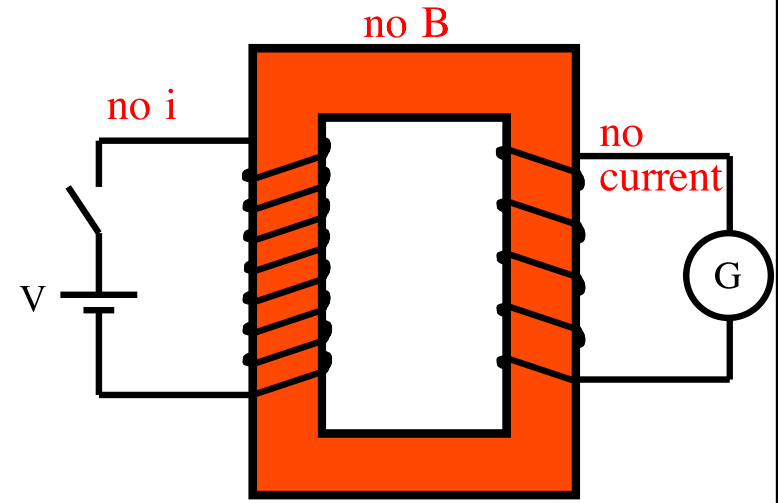


--As the B-fld decreases, an induced EMF is set up in the secondary coil opposite the original producing an induced current in the secondary coil.

--The induced current in the secondary coil WILL CEASE once the current in the primary has dropped to zero, as the *B-fld* in the yoke will cease to change at that point and there will no longer be a *changing magnetic flux* through the secondary coil.

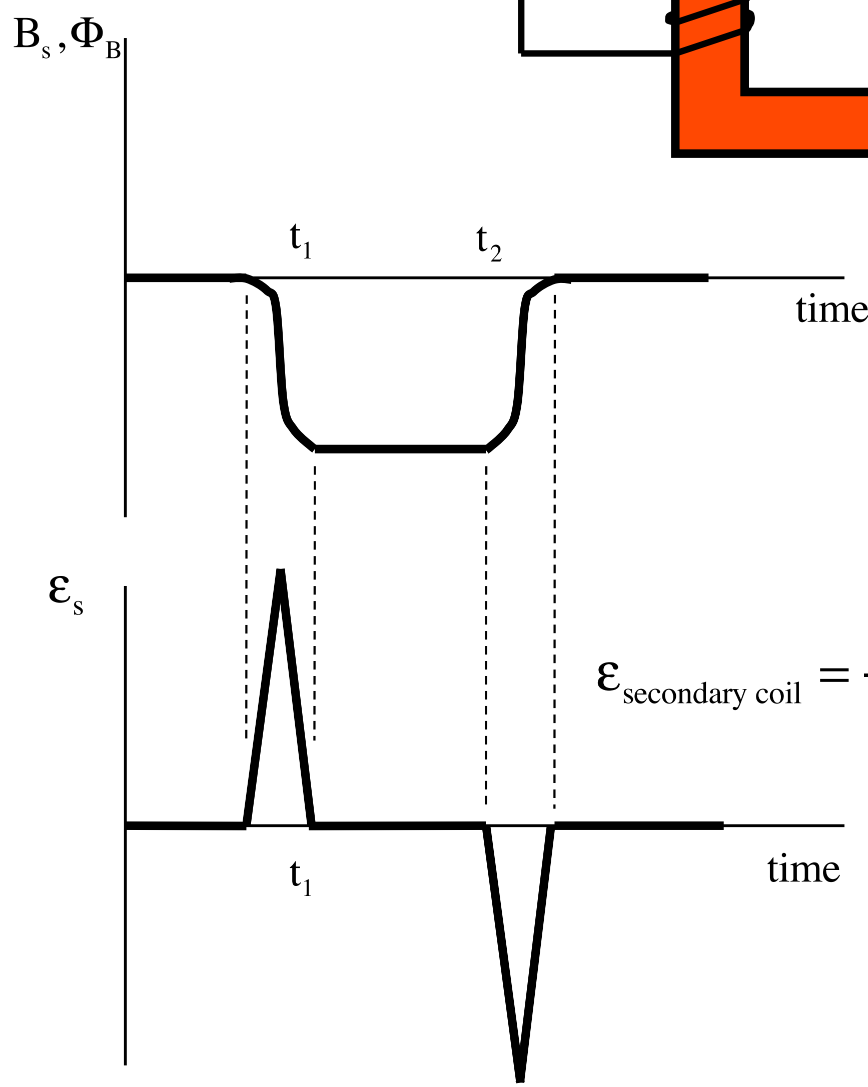


A rudimentary graph of the proceedings from the closing to opening of the switch:



--The first graph animates the *B-fld* and magnetic flux down the axis of the primary coil;

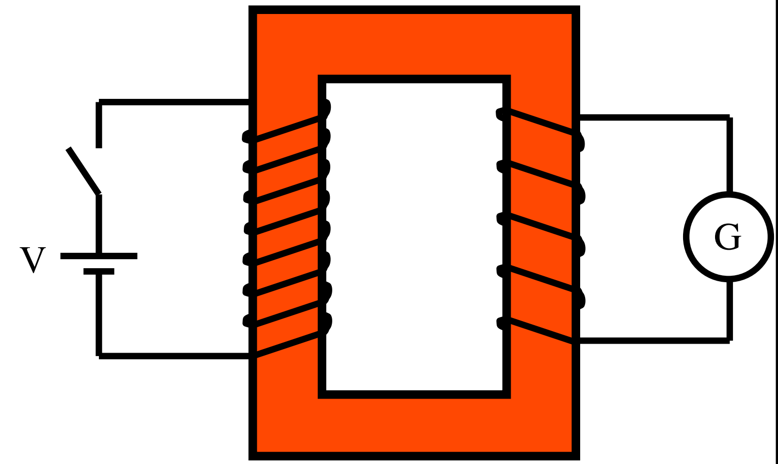
--The second graph animates the EMF induced in the secondary coil when current in the primary changes.



$$\varepsilon_{\text{secondary coil}} = -N_s \frac{\Delta\phi_B}{\Delta t}$$

What does the math suggest?

--Transformers transfer power from the primary coil to the secondary coil via a *changing magnetic flux* generated via a shared *B-fld.* That means we could write:



for the primary coil: $\varepsilon_{primary} = -N_p \frac{\Delta\phi_B}{\Delta t}$

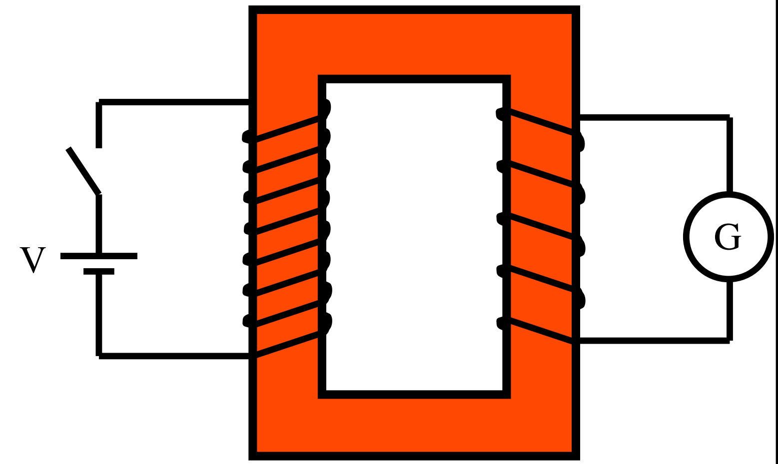
and for the secondary coil: $\varepsilon_{secondary} = -N_s \frac{\Delta\phi_B}{\Delta t}$

Taking the radio yields:

$$\frac{\varepsilon_{secondary}}{\varepsilon_{primary}} = \frac{-N_s\left(\frac{\Delta\phi_B}{\Delta t}\right)}{-N_p\left(\frac{\Delta\phi_B}{\Delta t}\right)}$$

$$\Rightarrow \quad \frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}$$

This ratio suggests that if you have more winds in the secondary coil, you will end up with a larger EMF in the secondary coil . . . You will have *stepped the voltage up,* so to speak.



This kind of transformer is called a *step-up* transformer. It's characteristic is that:

$$N_s > N_p$$

You never get something for nothing, though. What is being transferred is *power*, and assuming ALL the power is transferred with no loss, we could write:

$$P_{seconary} = P_{primary}$$

$$\varepsilon_s \; i_s = \varepsilon_p \; i_p$$

$$\left(N_s \frac{\Delta\phi_B}{\Delta t}\right) i_s = \left(N_p \frac{\Delta\phi_B}{\Delta t}\right) i_p$$

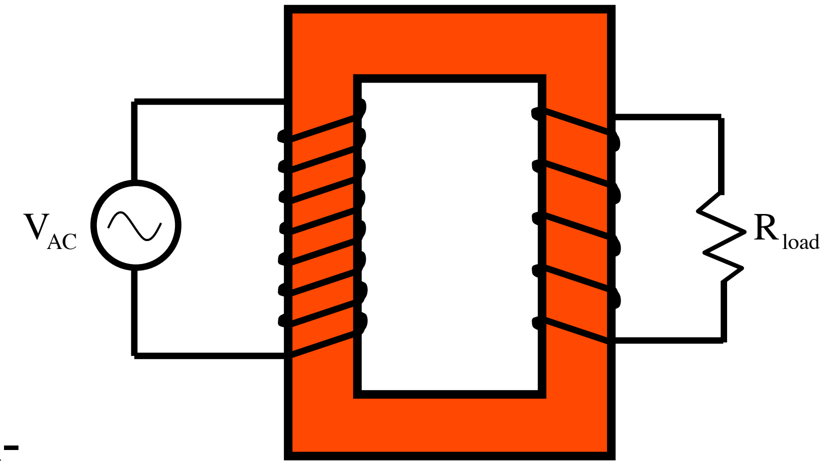
$$\Rightarrow (N_s) i_s = (N_p) i_p$$

$$\Rightarrow \frac{N_s}{N_p} = \frac{i_p}{i_s}$$

Translation: If the voltage goes up in a step-up transformer, the current provided to the secondary coil must go *down*.

NOTE: Using a transformer in a DC setting is nonsensical. The only time you get action in the secondary is when you change something in the primary. But using it with an AC source and ahhhhh, that's when you get poetry.

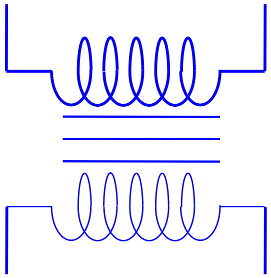
Summary:



--Assuming you're using AC, transformers allow you to transfer power from one part of an electrical circuit to another without electrically connecting the parts. It does it by utilizing two coils that are not electrically connected, but that share a common magnetic fld.

--Manipulating the *winds ratio* allows you to step-up the voltage or step-down the voltage from a source. This means that almost every electrical device you use has a transformer in it. (Example: the motherboard of a computer requires between 2 and 5 volts, but an AC wall socket is rated at 120 volts. The first thing your power cord runs into when it enters a computer is a transformer that steps the voltage down to a useable rating.)

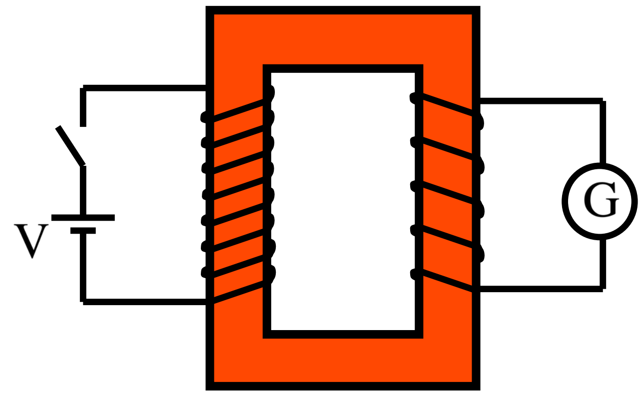
--And FYI, the symbol for a transformer in a circuit is shown to the right (it's supposed to signify two coil that aren't connected to one another, coupled by a magnetic field signified by the three lines between the coils):



Take-home message

So, if the winds ratio is such that there are more winds in the primary coil, then:

$$N_p > N_s \text{ and } \frac{N_p}{N_s} = \frac{\varepsilon_p}{\varepsilon_s} = \frac{i_s}{i_p}$$



In that case, the induced EMF in the secondary coil will be **less** than the induced EMF in the primary coil; the induced current in the secondary coil will be greater than the induced current in the primary coil, and the transformer itself is termed a “step down” transformer as that is what is happening--the voltage between the primary and secondary circuits is STEPPED DOWN.

If the winds ratio is such that there are more winds in the secondary coil, then:

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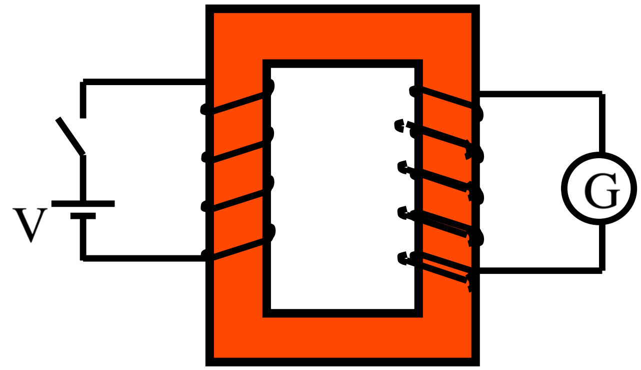
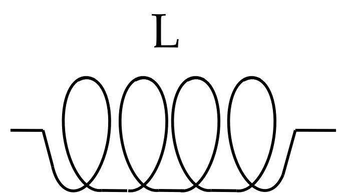


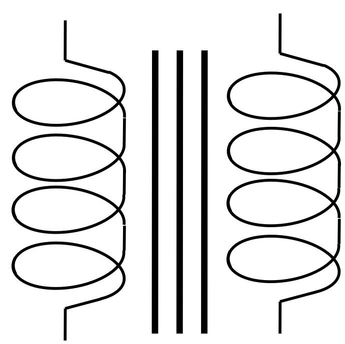
In that case, the induced EMF in the secondary coil will be greater than the induced EMF in the primary coil; the induced current in the secondary coil will be less than the induced current in the primary coil, and the transformer itself is termed a “step up” transformer as that is what is happening--the voltage between the primary and secondary circuits is STEPPED UP.

Circuit terminology

The electrical symbol for an inductor (a single coil) of inductance "L" is (if we haven't talked about coils in circuits yet, don't worry, we will):



The electrical symbol for a transformer is two coils side by side, not electrically connected by coupled magnetically (shown by three lines between them). See below:



Example problem

- You've gone to Europe. You've taken your favorite hair dryer, which runs on 110 volt AC. The Europeans use 220 volt AC, so you must use a transformer to adjust the power to 110 volts.
  - What kind of transformer do you need to use?

    Step down (from 220 V to 110 V)

  - If your secondary coil has 84 coils, how many does the primary coil have?

    $$\frac{V_p}{V_s} = \frac{N_p}{N_s} \text{ so } N_p = (84\ coils)\frac{(220\ V)}{110\ V} = 168 \text{ coils}$$

  - Let's assume your hair dryer draws 3 amps. Which circuit, the primary or the secondary, must have 3 amps in it?

    The secondary must have 3 amps

  - How much current will there be in the primary coil if the secondary has 3 amps flowing through it?

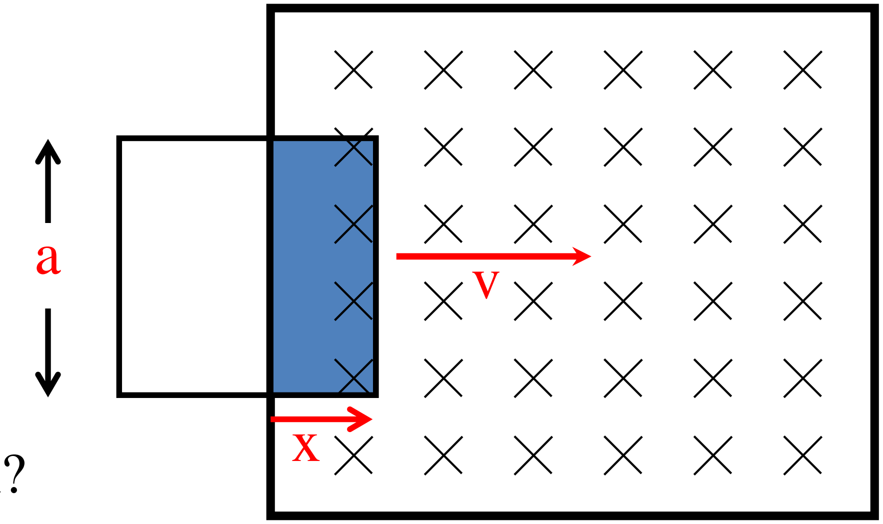
    $$\frac{N_p}{N_s} = 2 = \frac{i_s}{i_p} \text{ so } i_p = \frac{3A}{2} = 1.5A$$

Review of Motional EMF

- Bottom line: Any coil that moves into or out of an external magnetic field will feel a magnetic force that fights the motion.
  - That is, if the coil is pushed *into* the magnetic field, the force produced by the interaction of the induced current in the coil with the external field will make it harder to do this.
  - If the coil is pulled *out of* the magnetic field, the force produced by the interaction of the induced current in the coil and the external magnetic field will make it harder to do this.
- IN ALL CASES, the interaction of the induced current with the external magnetic field will produce a force on the coil that FIGHTS THE CHANGE.

So Back to Motional EMFs

Example 4: *A square coil* of resistance *R* and sides of length *a* enters a region in which there is a constant *B-fld*. It is moving with constant velocity *v* as shown in the sketch:



a.) Is there a magnetic flux through the coil?

yes, magnetic field lines are piercing the face of the coil

b.) Is there an induced EMF set up in the coil (justify)? If so, what is its magnitude?

yes, the magnetic flux is CHANGING through the face of the coil

What was the technique to determine the EMF? Define the magnetic flux, then take it's derivative! With N = 1:

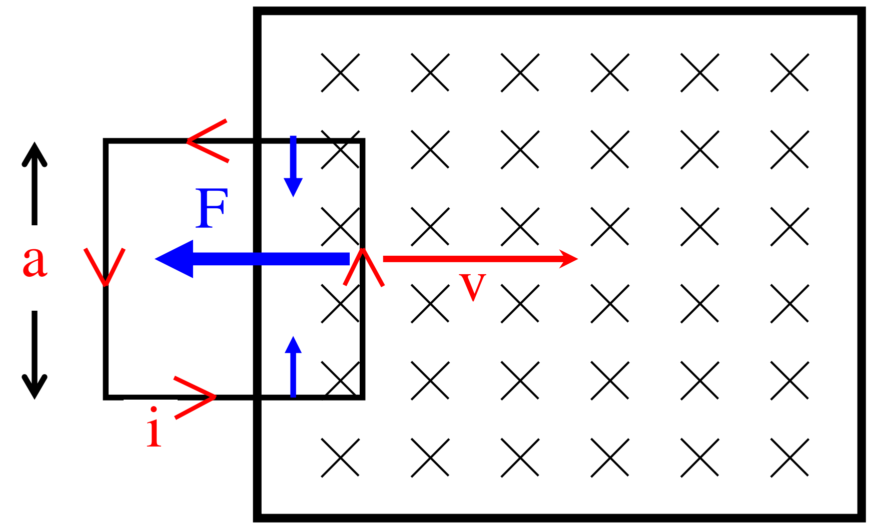
$$\varepsilon_{ind} = -N\frac{d\Phi_B}{dt}$$

$$= -\frac{d(Bax)}{dt}$$

$$= -Ba\frac{dx}{dt} \qquad (-Bav)$$

c.) What is the induced current in the coil?

$$i_{ind} = \frac{\varepsilon_{ind}}{R}$$
$$= \frac{Bav}{R}$$



d.) What is the direction of the current?

Lenz's Law:

--external *B-fld* into the page;

--magnetic flux increasing,

--so induced *B-fld OUT OF PAGE* (opposite external field). Current has to flow counterclockwise to achieve that.

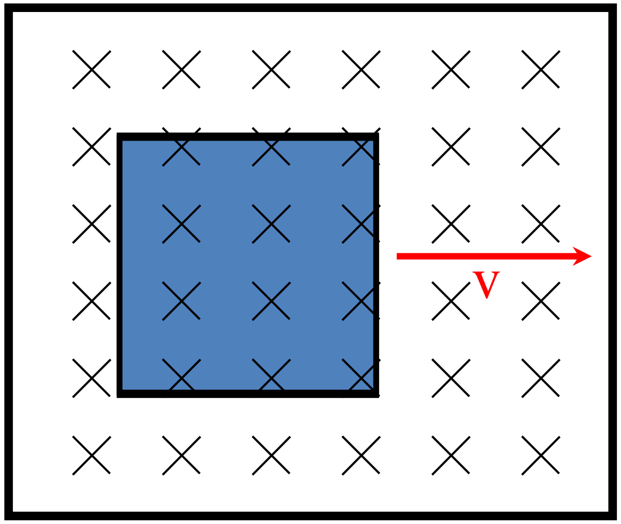
e.) The induced current will interact with the external *B-fld* and feel a force. In what direction will be that net force?

The magnitude would be the magnitude of $\vec{F}_{wire} = i\vec{L}x\vec{B}$, which we could figure out, but all that was asked for was the direction, which is the direction of that cross product. The force on the two horizontal wires will cancel, but the force on the vertical wire in the *B-fld* will be to the left, as shown on the sketch.

The coil proceeds into the *B-fld*, fully immersing itself. At that point:

f.) Is there a magnetic flux through the coil?

yes, magnetic field lines are piercing the face of the coil

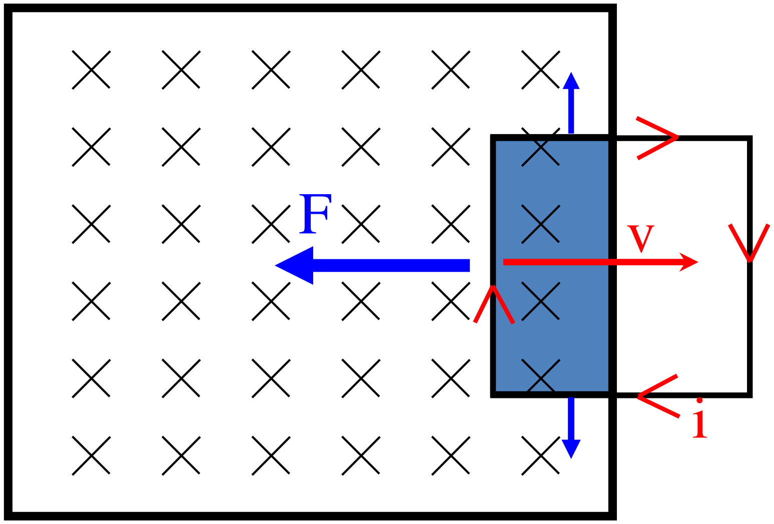


g.) Is there an induced EMF set up in the coil (justify)? If so, what is its magnitude?

Nope, the magnetic flux is NOT CHANGING through the face of the coil, so there is no induced EMF set up in the coil.

And that means there's *no induced current* and *no magnetic force* acting to fight the motion of the coil as it moves through the field (there will be that dipole, but it won't retard the motion).

The coil proceeds out of the *B-fld*, leaving it with time. At the point shown:



h.) Is there a magnetic flux through the coil? *yes,* magnetic field lines are piercing the face of the coil

i.) Is there an induced EMF set up in the coil (justify)? If so, what is its magnitude?

yes, the magnetic flux is CHANGING through the face of the coil. We won't do the calculation—it will be similar to what we did earlier—but we could.

j.) What is the direction of the induced current?

Lenz's Law maintains clockwise (go through the steps for the practice).

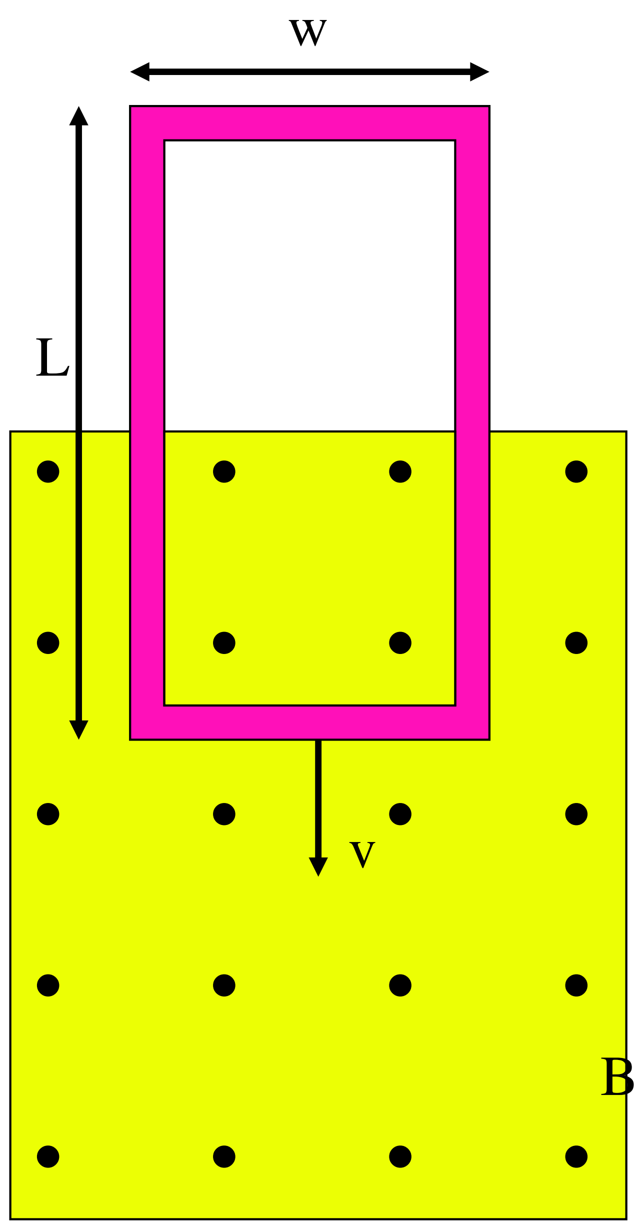
e.) The direction of the induced force on the coil?

$\vec{F}_{wire} = i\vec{L}x\vec{B}$ Says the vertical wire will feel a force *to the left* (again—do it!).

Huge observation: Induced currents will ALWAYS generate forces that fight what you are trying to do. Try to move the coil OUT OF THE FIELD—the induced force will fight you. Try to move the coil INTO THE FIELD—the induced force will fight you . . . they always *fight the change*.

Problem 20.67

- A loop of mass m, resistance R and dimension w and L falls from rest into a B-field as shown. During the time interval before the top edge of the loop reaches the field, the loop reaches terminal velocity.
  - a.) What's terminal velocity?
  - b.) Why is the terminal velocity proportional to R?
  - c.) Why is the terminal velocity inversely proportional to B^2?



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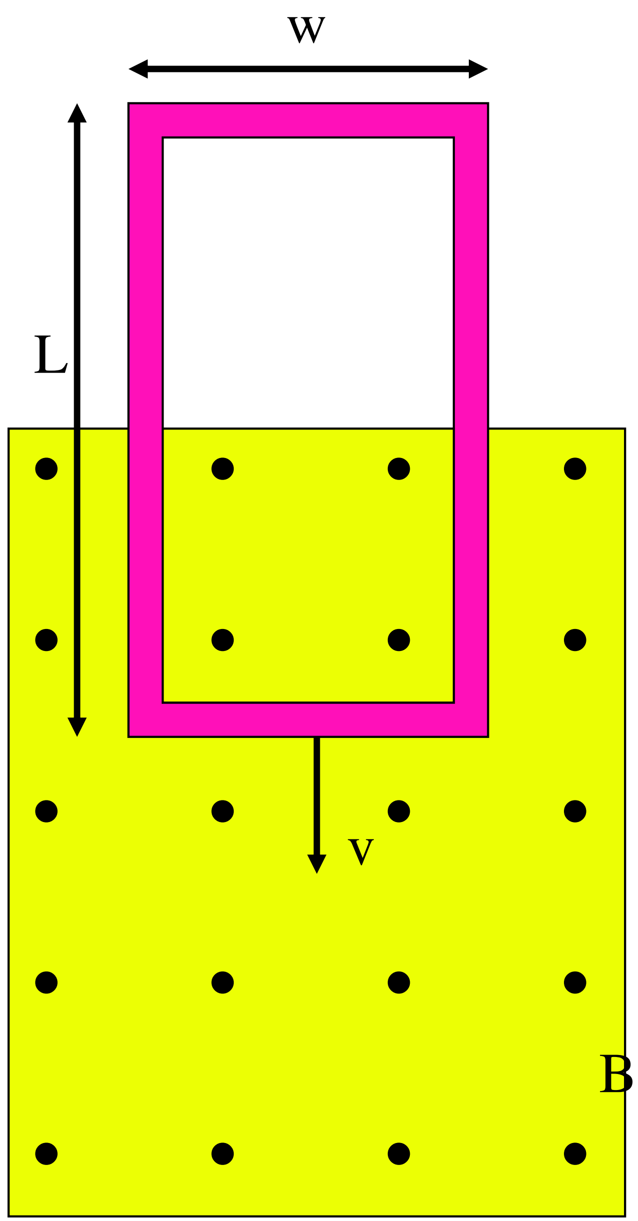


a.) What's terminal velocity?

Terminal velocity occurs when the force of gravity is exactly counteracted by the induced force generated by the induced current in the coil interacting with the external magnetic field. That is:

$$\sum F_y :$$

$$i_{induced} wB - mg = m\cancel{a_y}^{=0}$$

To get the induced current, we have to used Faraday's Law and Ohm's Law in conjunction with one another.

In other words, with N=1 we can write:

$$i_{induced} = \frac{|\varepsilon_{induced}|}{R}$$

$$= \frac{\left|-N\frac{\Delta\Phi_B}{\Delta t}\right|}{R}$$

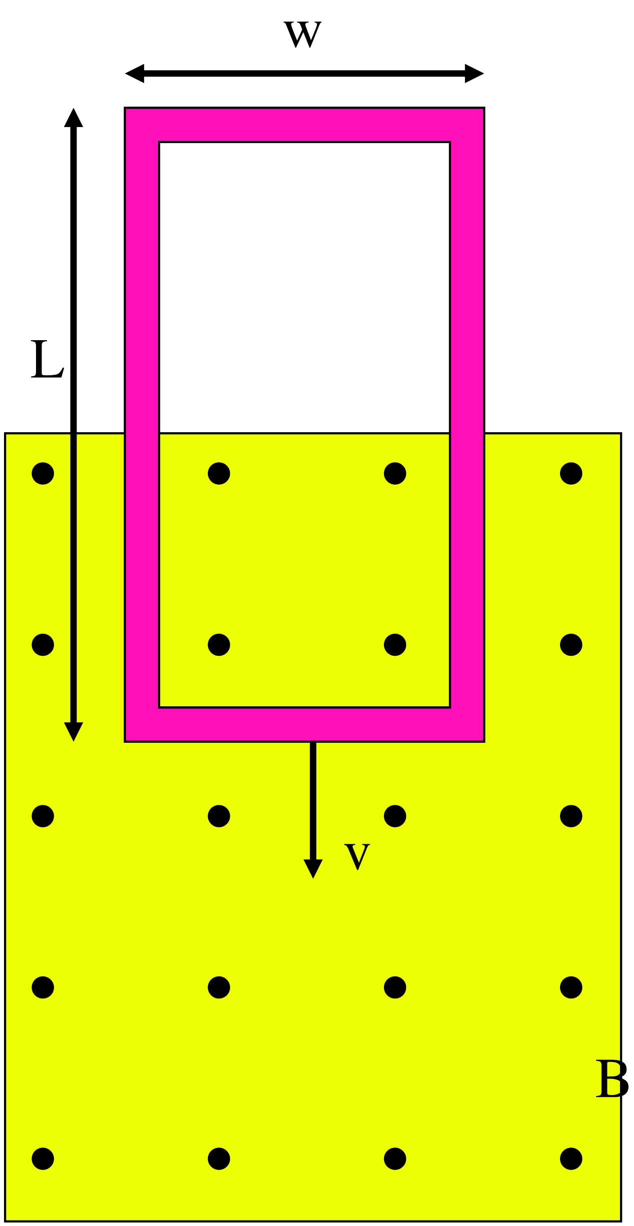
$$= \frac{NB\frac{\Delta A}{\Delta t}cos0°}{R}$$

$$= \frac{(1)[\frac{w(x+\Delta x) - wx]}{\Delta t}}{R}$$

$$= \frac{Bw\frac{\Delta x}{\Delta t}}{R}$$

$$i_{induced} = \frac{Bwv_{terminal}}{R}$$

Why is the - sign not important here?

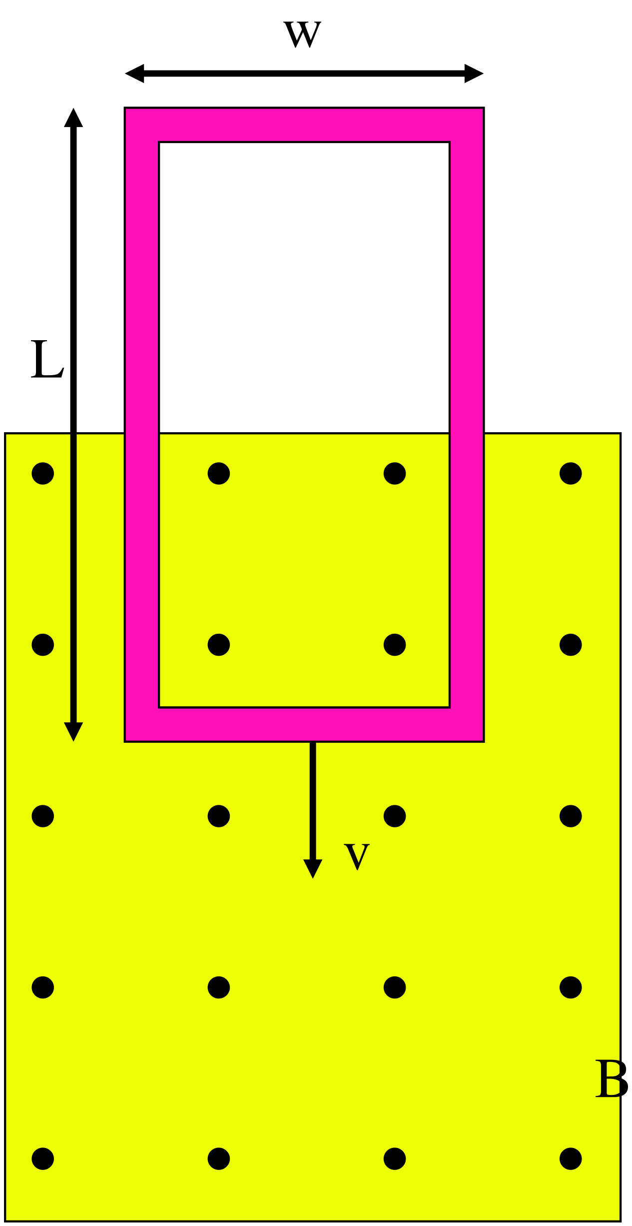


Coupling the two equations

$$i_{induced} = \frac{Bwv_{terminal}}{R} \qquad i_{induced}wB - mg = m\cancel{a}_y^{=0}$$

we can write:

$$i_{induced}wB - mg = m\cancel{a}_y^{=0}$$
$$\Rightarrow \left(\frac{Bwv_{terminal}}{R}\right)wB = mg$$
$$\Rightarrow v_{terminal} = \left(\frac{mgR}{B^2w^2}\right)$$



b.) Why is the terminal velocity proportional to R?

c.) Why is the terminal velocity inversely proportional to B^2?

b.) Why is the terminal velocity proportional to R?

Bigger resistance R means smaller current i. Smaller current means less magnetic force on the wire, which means gravity holds sway and the maximum velocity is greater. That is, v is proportional to R.

c.) Why is the terminal velocity inversely proportional to B^2?

If the magnetic B gets bigger, you'd expect the current interaction with the B-field to be larger and the terminal velocity to be smaller. That means the terminal velocity and magnetic field are inversely proportional.